

P425

PRINCIPAL MATHEMATICS

A LEVEL MATHS

**S.5 MATHEMATICS
COMPETENCE BASED ITEMS**

HOLIDAY PACKAGE 2025

TERM TWO

COMPILED BY,

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Item 1

A local environmental group in Jinja is studying the population growth of a specific fish species in a protected section of the Nile River. Their initial estimate in 2020 was 1,200 fish. They believe the population P after t years can be modelled by $P(t) = 1200 \times (1.15)^t$. However, another model proposed is based on logarithms; $\log_{10} P = \log_{10} 1200 + t \log_{10} 1.15$.

Tasks:

- (a) Using the index model $P(t) = 1200 \times (1.15)^t$, Calculate the predicted fish population in the year 2025.
- (b) Using the logarithmic model, show that it is equivalent to the index model.
- (c) The group wants to know when the fish population is predicted to reach 5,000. Using logarithms and the model $P(t) = 1200 \times (1.15)^t$. Determine the approximate number of years (t) it will take.

Item 2

A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B. Let the costs of drilling boreholes A, B and C be a , b , and c in UGX respectively.

Tasks:

- (a) Formulate a system of three linear simultaneous equations representing the information given.
- (b) By Using Row reduction, calculate the individual cost of drilling each borehole (a , b and c).
- (c) If the cost per meter drilled was UGX 250,000 for all boreholes, determine the depth of Borehole A.

Item 3.

An economist is studying the relationship between investment I and national income Y . The relationship involves a complex function where a particular term is given by $h(Y) = \frac{Y^3 + 2Y^2 - Y + 5}{Y^2 + Y - 2}$.

Before proceeding with the economic analysis, the economist needs to simplify this expression.

Tasks:

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Help the economist to:

- (a) Identify $h(Y)$ as a proper or improper rational function. Justify your answer.
- (b) Express $h(Y)$ as the sum of a polynomial and a proper rational function.
- (c) Take the proper rational fraction part obtained in (b) and decompose it into its partial fractions.
- (d) Combine the results from (b) and (c) to write the complete simplified expression for $h(Y)$.

Item 4.

A bus heading to Masaka passes through Mpigi town.

At time, t hours, past noon, the distance of the bus from Mpigi to Masaka is expressed as $x = 7\sin t - 4\cos^2 t + 2$. It's known that 1 degree is $\frac{\pi}{180}$ rad.

Task: Help:

- (a) The manager wants know when the bus was in Mpigi give the answer to 2 decimal places
- (b) Identify the time the bus will be behind Mpigi by a distance of 5km.

Item 5.

A group of learners were taken to a certain factory in Jinja to fully understand the concept of probabilities. The factory produces light bulbs using three machines: Machine A, Machine B, and Machine C.

Machine A produces 40% of the total output, and 5% of its bulbs are defective.

Machine B produces 35% of the total output, and 3% of its bulbs are defective.

Machine C produces 25% of the total output, and 2% of its bulbs are defective. A bulb selected at random from the factory's output.

Later, these learners were given asked to use the knowledge to also analyse the given problem.

Two boxes M and N contain 4 red and 3 blue pens and 3 blue and 4 red pens respectively. A box is selected at random and from it a pen is drawn and placed into the second box, the second box is shaken and from it the pen is drawn and placed into the first box.

Task:

- (a) Determine the probability that the selected bulb was produced by Machine A and is defective.
- (b) Given that the selected bulb is found to be defective, calculate the probability that it was produced by machine B.

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- (c) Determine the probability that the pen selected from the first box is red.

Item 6.

A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, $\angle PQR$, is measured as 75° . The surveyor needs to find the length of the third side QR and the area of the land.

The Surveyor also had to test whether its true that

$$\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}.$$

Task.

- Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.
- Calculate the area of the triangular piece of land PQR.
- If angle QPR was actually measured as $(45^\circ + 30^\circ)$, apply an appropriate formula to find the exact value of $\cos 75^\circ$.
- Is it true that $\cos 18^\circ$ is equivalent to the given surd? Justify.

Item 7

Two children are pulling a toy wagon across a smooth playground surface using two ropes. One child pulls with a force of **12N**, and the other pulls with a force of **9N**. The ropes make an angle of **60°** with each other. As a result of their combined effort, the wagon starts to move with an acceleration of **3.65 m/s^2** .

Task

Determine the mass of the wagon

Item 8

A nursery school teacher wants to understand whether there is a relationship between her pupils' **reading** and **writing** abilities. She believes that pupils who perform well in reading may also perform well in writing, but she wants to use data to confirm this before designing a combined reading-writing support program. She records the following scores for **seven pupils**:

Pupil	A	B	C	D	E	F	G
-------	---	---	---	---	---	---	---

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Reading Score	63	81	73	23	33	41	53
Writing Score	77	69	84	61	58	62	69

Task:

Help the teacher to understand the relationship between reading and writing

Item 9

A delivery worker places a 1 kg parcel on a rough ramp inclined at 30° to the horizontal. The coefficient of friction between the parcel and the ramp is 0.25. To avoid any accidents, the worker needs to apply a force parallel to the ramp to keep the parcel from sliding.

Task

Determine the minimum force that the worker must apply to just prevent the parcel from sliding down

Item 10

A farmer is measuring a rectangular plot of land for planting. The measured dimensions are **1.25 km** by **0.44 km**.

Task

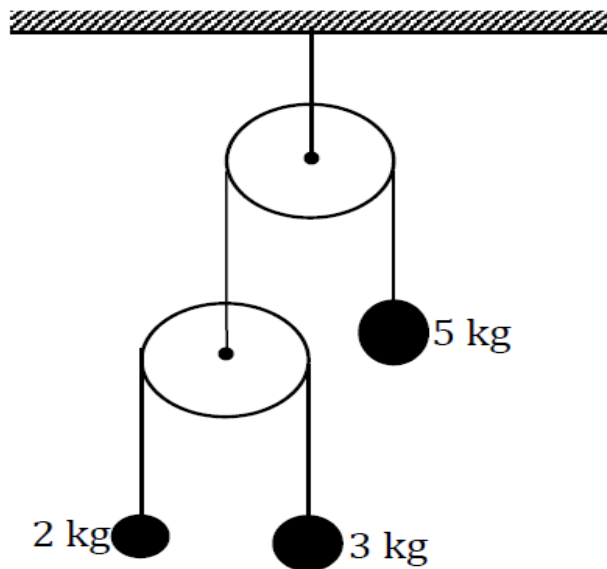
:

Enable the farmer to know the limits within which the area of the plot lies, based on the given measurements.

Item 11

During a school project, a science club sets up a compound pulley system to study motion and tension. In their set up, a light inextensible rope passes over a smooth fixed pulley A. One end holds a 5 kg toolbox, while the other end is attached to a smooth pulley B. Another rope runs over pulley B, supporting two metals of 3 kg and 2 kg.

Support;



Task :

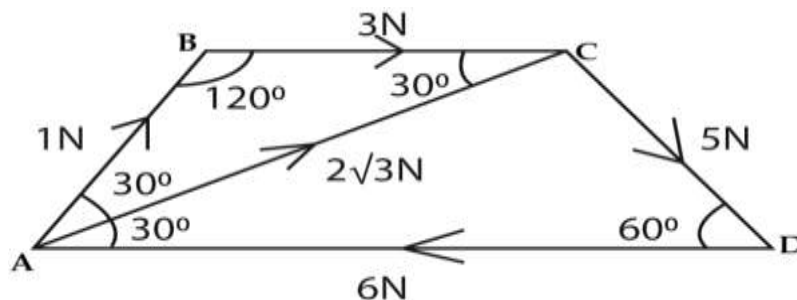
- Help the club to determine the acceleration of pulley B
- Enable the club members know the tension in each rope.

Item 12

A technician is testing a metal frame shaped like a trapezium **ABCD**, which is part of a foldable stand for holding a solar panel. The lengths of the sides are: **AB = 2 meters**, and **AD = DC = CB = 1 meter**.

To check how strong the frame is, different forces are applied to it. These forces have magnitudes of **1 N**, **3 N**, **5 N**, **6 N**, and **$2\sqrt{3}$ N**.

Support:



Task

Help the technician to determine the resultant force on the frame

Item 13

A teacher wants to find out if there is a relationship between how students perform in **Math** and **English**. She collects test results from **10 children** in her class, as shown below:

Child	A	B	C	D	E	F	G	H	I	J
Math Marks	1	8	15	18	23	28	33	39	45	45
English Marks	3	14	8	20	19	17	36	26	14	29

Task:

- Using a scatter diagram, help the teacher to understand the relationship between Math and English marks.
- Estimate the Math mark of a student who scored 30 marks in English.
- Calculate the rank correlation coefficient to measure the relationship between the two subjects and comment on your results using the 5% level of significance.

Item 14

A veterinary officer recorded the weights of animals brought to a community clinic over one month. The animals were grouped by weight, and the number in each group was recorded as shown below:

Mass (kg)	21–25	26–30	31–35	36–40	41–50	51–65	66–75
Frequency	10	20	15	10	30	45	5

Task:

- Help the veterinary officer to represent the data on a histogram and use it to estimate the mass with the highest number of animals.
- Determine the median for the above data

Item 15

The remote village of Kijungu relies heavily on its local well for water due to climate change and increasing population, the well's water output is becoming very low, an engineer has developed a model to predict the well's daily water output in liters based on the factor x which accounts for drought conditions

increased demand and underground water table depletion, the model is given by the equation $2^{2x+1} + 2^{x+1} + 1 = 2^x$.

Task.

As a mathematics students help in identifying the stress factor x at which the wells water output meets the condition described by the above equation.

Item 16

A certain farmer in the village has bought land which he plans to fence , he has contracted a person to fence it and the contractor has been informed that the area of the land is $6300m^2$. this land is rectangular in nature and that the length is $20m$ more than the width .

Task.

(a).by completing the squares determine the actual dimensions of the peace of land.

(b) determine the maximum value of the expression obtained from above.

Item 17

A certain city is experiencing social challenges with two distinct population groups having different experiences of urban life, the city planners are trying to understand if there is a common problem affecting both groups, they are modelling the wellbeing of two groups using two separate quadratic equations. $x^2 + px + q = 0$ p represents access to social services and q represents employment opportunities.

and $x^2 + mx + k = 0$. The collective wellbeing is given by , where m represents educational attainment and healthcare access, k represents safety quality.

The social challenge exists if $(q - k)^2 = (m - p)(pk - mq)$.

Task.

As a data analyst identify whether the common social challenge exist or not.

Item 18

A certain city in Uganda is investing in a new smart traffic management system designed to optimize traffic flow and reduce congestion especially during the rush hours. The system caters road density, vehicle capacity and pedestrian movement, the engineers have developed some indicator related to

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the efficiency of the traffic flow on the major roads. The indicator θ , which relates to a specific traffic cycle parameter which is considered when the following equation is satisfied. $4 + \tan^2 \theta = \sec \theta (7 - \sec \theta)$. The system needs to be designed in way such that an optimal traffic flow is achieved.

Task.

Help to determine the indicator values (θ) at which the system may be efficient.

Item 19

The city is facing a challenge in urban planning where by balancing economic development and preservation of green spaces, to model this relationship between economic development and preservation of the green space, the city planner comes up with a simplified systems of logarithms with two indicators x and y , he believes that for optimal urban health, the indices should satisfy the following two logarithmic relationships.

$$\log_{10} x - \log_{10} y = 1.0 \text{ and } \log_{10} x + \log_{10} y = \log_{10} 2.5$$

Task

Use the above system of simultaneous equations to obtain the indicators of sustainability economic development

Item 20

Your designing a pulling mechanism for a new agriculture machine in Fort portal, and it has been observed that when two people apply a force of magnitudes 3N and 7N inclined at an angle of 60° to each other, the agriculture manager of the city is uncertain of the single force that can be used to move the machine.

Task.

Help the officer to determine the single force that can be used to move the machine and the direction.

Item 21

A treasure map uses a coordinate grid to help adventures locate hidden treasures in Kassanda district. The grid is labelled with coordinates (x, y) where each point represents a specific location. Two equations are used for

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safety of the treasures but the travelers have no knowledge of how to identify the coordinates. These equations are $5^{x+2} + 7^{y+1} = 3468$ and $7^y = 5^x - 76$ Task Help them to identify the location of the treasures.

Item 22

Your class teacher loses her phone after being stolen by thugs while coming to school. He decides to go to police so that the phone can be tracked using GPS coordinates. The phone's location at different times is recorded and the police are trying to find its movement pattern. The tracking device uses a logical operation given simultaneously as $2^x + 3^y = 5$ and $2^{x+3} - 3^{y+2} = 23$. When they got so close to the phone, they could not tell the coordinates to locate the phone with ease. The coordinates are in meters and one point gives the position of those with the tracking device, the other point gives the position of those with the phone.

Task

- (a) Help them find the coordinates.
- (b) How far is the phone from the police tracking it?

Item 23

A robot arm moves such that the positions of its ends are described by $x = 7 + \sqrt{3} \cos \theta$ and $y = 5 + 7 \sin \theta$ where θ is the rotational angle. The robot operator needs to determine the largest reflex angle of rotation of the robot arm if it is known that $x + y = 13$.

Task

- (a) Show that $49x^2 + 3y^2 = 686x + 30y - 2329$
- (b) Help the robot operator get the required rotational angle.

Item 24

The number of items (y) produced by a company are modelled by the equations

$y = 8 \sin 2x - 5 \cos 2x$ where x represents number of production inputs needed for the company to maximize production.

Task

Help the company manager find out the smallest number of inputs(x) needed for the company to maximize production.

Item 25

Mutebi a boda-boda rider in Kampala wants to buy land that is expected to be worth *shs2 million* at the end of the year 2025. He has decided to start saving *shs 2,0* on day 1, increasing the amount by *shs 500* every day. However, he is not sure whether he will be ready to buy this land by the end of *160 days*. He wants to tighten his spending on food if by this time, he will not be able to fulfil his goal.

Task

Help Mutebi find

- (a) Whether his savings will be ready by the stated time.
- (b) Number of days needed to raise *shs 12 million*.

Item 26

Alice, a farmer in Mbale district is planning to plant sorghum in parallel lines. The first proposed line is to pass through the earth coordinates *(10, 50)* and *(30,)*, while the second line is to pass through the earth coordinates *(15, 60)* and *(25, 40)*. However, she is not sure whether these lines are actually parallel to each other. to ensure access to sunlight a reasonable distance between these lines is needed. The minimum distance needed between them is *100 cm*, but Alice does not know if the given coordinates do not meet this requirement.

Task

Help Alice find

- (a) The equations of the first and second lines. Convince Alice that these lines are surely parallel to each other.
- (b) Whether Alice should increase the distance between these lines

Item 27

A local health clinic is conducting a study on the prevalence of obesity and malnutrition in a community. As part of their assessment, they measured the

masses (in kilograms) of 40 individuals randomly selected from the community. The data collected is as follows:

46 52 62 55 61 48 57 46 70 60 54 49
 47 52 48 52 60 55 50 53 64 54 54 53 57
 58 51 64 56 61 52 58 41 59 57 44 51
 58 68 65

The clinic is particularly concerned about identifying mass ranges that might indicate a higher risk of health issues within the community.

Task.

Help the clinic to:

- Construct a frequency distribution table with equal class intervals, beginning with the 41-45 kg class.
- Determine the expected mass.
- calculate the standard deviation of the above given information
- using an appropriate diagram, determine
 - the number of individuals above 50kg mass.
 - the middle 40th percentile.

Item 28

A school administrator at a certain school want to analyses the performance of 10 students in two subjects, mathematics and physics, to identify any potential relationships between their scores, the following table shows the marks obtained by these 10 students

Student	1	2	3	4	5	6	7	8	9	10	
Mathematics		40	90	54	32	80	65	55	48	55	30
Physics	68	40	47	64	55	41	62	76	74	80.	

Task

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- (a). draw a scatter diagram to represent the data above, based on the trend comment on the relationship observed between the student's marks.
- (b). draw the line of best fit and use it to estimate the mark a student who scored 60 in mathematics would likely get in physics.
- (c). calculate the rank correlation between mathematics and physics, what does this coefficient suggest about the relationship between performance in these subjects?
- (d). using your results in (c) above comment at 1% and 5% level of significance.

Item 29

An agriculture engineer is designing an automated sprinkler for a new precisely shaped crop garden, the field is perfectly hexagonal with a central pump station and the main distribution pumps and the main distribution pumps extended to the vertices, from each vertex a sprinkler is installed along the sides of the hexagon, the efficiency of the sprinkler system depends on the force exerted by each sprinkler head, the forces acting along the sides of the hexagon include the following, 4N, 2N, 10N, 1N, 7N, 3N, and $2\sqrt{3}N$ all the forces are indicated in the directions represented by letters AB, BC, CD, DE, EF, FA, and AD, the engineer is uncertain of the single force that can be used to pump the Water.

Task

Help the engineer in determining the single force that can be used to pump the water, hence determine its direction of the resultant force

Item 30

A team of engineers is trying to move a 5kg portable generator on a rough inclined plane, the incline is at 35° to the horizontal, to prevent the generator from

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from sliding down, a horizontal force of 20N is applied. the body is found to be on the point of moving up the plane,

Task

Determine the coefficient of friction between the generator and the plane.

Item 31

A logistic manager at a local depot in fort portal needs to move a 200kg box of electronics across a rough concrete floor to a new storage location, the coefficient of static friction between the box and the floor is 0.5. to minimize the effort and prevent damage, a worker will use a light rope attached to the box to pull it, the manager needs to know the minimum tension required in the rope to just get the box moving under two different conditions.

The worker pulls the box at angle of 30° above the horizontal

The worker pulls the box at an angle of 30° below the horizontal,

Task

Determine the tension in the string in both cases and advise the manager on the most efficient method to use when pulling the box

Item 31.

The National Water and Sewerage Corporation (NWSC) is designing a water distribution system for three neighboring communities in Kampala. Each community has different water requirements and infrastructure constraints. A consultant engineer working on his project wants to determine the optimal flow rates for each community.

The water distribution system is modeled by the following equations

$$X + 2Y + Z = 2400$$

$$2X + Y + 3Z = 3900$$

$$3X + 4Y + 2Z = 5100$$

Where X, Y and Z represent the flow rate in liters per minute in communities A, B and C respectively

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The polynomial $P(X) = X^3 - 7X^2 + 14X - 8$ models the operational efficiency of the pumping systems

Tasks

- a) Help the engineer to determine the optimal flow for each community by use of echelon row reduction.
- b) if the community A's water requirements increase by 200litres per minute. What adjustments should be made to other communities' supply to maintain the system balance?
- c) Determine all possible values of X where the efficiency of the pumping system is zero.

Item 32

In the mathematics contest at Makerere University, the board of examiners set four problems which are to be solved by the learners but one of the learners said that the problems had errors since she failed to come out what is required

Problem 1: if $y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$, then $\frac{dy}{dx} = 1$

Problem 2: if $y = \tan^{-1} x$, then $(1 + x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$

Problem 3: if $y = \frac{\cos x}{x}$, then $\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} + y = 0$

Problem 4: if $y = Ae^{3x} + Be^{-x}$, then $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} - 2y = 0$

She was told that all problems.

Tasks

Help the student to prove the above problems.

Item 33

The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala -Jinja high way. They've developed a mathematical model to predict traffic density at different times of the day. The traffic density function at time t hours after 6:00 AM is given by;

$$D(t) = \frac{200x}{x^2 + 4x + 3}$$

Tasks

As a traffic engineer, you need to analyze this model to optimize traffic flow and plan future road expansions.

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(a)Representing the function as a sum of partial fractions.

(b)By partial fraction decomposition, determine the time of the day when traffic density is at its minimum or maximum.

Item 34

Mukisa had three numbers expressed as logarithms, His friend Tonny said the numbers forms an arithmetic sequence with the middle term a , and common difference, d . Tonny said that he formed an expression $ad^2 = a^3 - 1$. Given the numbers: $\log_z x, \log_y z, \log_x y$.

Mukisa's father is a city tycoon whom he invested \$100,000 in a business at a compound interest of 8% per annum. he planned to get money from the business after 10 years

Task

- i. As a mathematics student prove Tonny's expression if it holds for arithmetic sequence
- ii. Help Mukisa's father to know the amount of money he is to get after 10 years
- iii. What is the it takes to reach more than \$300,000.

Item 35

Mr. Lubwama is a mathematics teacher at Light secondary school; He was teaching about binomial theorem but later realized that he was to first teach permutations and combinations he introduced some part of it and gave out $20C_r = 20C_{r-2}$ in the classroom was trying to expand $\sqrt{\frac{1-x}{1+x}}$ using Binomial theorem. He told the students to use $X = \frac{1}{8}$ to determine the approximate value of $\sqrt{7}$

Task

- i. As a mathematics student, he the learners in the classroom to find the value of r

To find the approximate value of $\sqrt{7}$ by using the first three terms of the expansion.

Item 36

Malaria disease is the leading cause of death among children in Uganda, to help bring down the mortality rate among the children, Lira regional referral hospital through the ministry of health, have embarked on the vaccination of children aged two years and below. Recently, the hospital receives a consignment of this vaccines. Each child requires from **2 0 0 m g** to **4 0 0 m g** of the drug per dose to be effective and safe.

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A new automatic dispenser is being calibrated by the technician to dispense the vaccines. However, as the technician was explaining to the community how it works, he noted that the dispenser sensor fluctuates, therefore he had modeled an expression to help determine the amount of medicine dispensed per dose as,

$$Q(x) = x^2 + 10x + 180.$$

Where;

$Q(x)$ - the amount of medicine in milligram (mg) dispensed depending on the sensor adjustment level x , a non negative number

TASK.

As a mathematics student who attended this training, help the technician to;

- Form an inequality representing the safe dosage condition
- Solve the inequality to find the range of adjustment level x for which the dosage remains within the safe limit, so that he can advise the community effectively.

Item 37

A team of students is working on a digital irrigation system for a school project. They encountered different mathematical models in form of rational functions to represent pressure, flow rate and energy loss in the water pipes. To analyze these models, they need to break each expression into simpler fractions which are easier to use in computations. The team encountered the following expressions but they couldn't express them into simpler forms:

The function for water pressure along a straight pipe section is modeled as

$$\frac{2x^2 - 5x + 7}{(x-2)(x-1)^2}$$

The expression for the flow rate of water in a narrow pipe is given as

$$\frac{4x^2 + 3x + 1}{(x+2)^2}$$

The function representing energy loss around a bend in the pipe is $\frac{x^4 - x^3 + x^2 + 1}{x^3 + x}$

Task:

As a student of mathematics, help the team of students on how they can express the given functions into simpler fractions so as to analyze the models.

Item 38

An engineer is installing a solar panel on a building. The owner of the building insisted that to tap maximum solar energy, the panel should be slanted at an angle of $70^\circ \leq x \leq 90^\circ$ to the eastern direction. To ensure this, the engineer used the equation $\cos 2x = 4 \cos^2 x - 2 \sin^2 x$ for $0^\circ \leq x \leq 360^\circ$. But the owner of the building was not satisfied with the model equation by the engineer and suggested to the engineer to use the identity $\sin(x + \alpha) = P \sin(x - \alpha)$, where x and α are angles

Task

As a student of mathematics:

- Help the owner of the building to prove that if $\sin(x + \alpha) = P \sin(x - \alpha)$, then $\tan x = \left(\frac{P+1}{P-1}\right) \tan \alpha$. Hence solve the equation $\sin(x + 20^\circ) = 2 \sin(x - 20^\circ)$ for $0^\circ \leq x \leq 360^\circ$.
- By solving the equations above for both engineer and the owner of the building, whose equation should be used during the installation of the panel and at what angle?

Item 39

At your school, the agriculture club of the school is planning to sell their poultry (one month old birds) to their customers. This is the third sales the club is making since they embarked in this business, from the previous market experience, they observe that;

As they reduce the selling price, more customers buy birds, increasing their total revenue. Their revenue function $R(x)$ in thousands shillings, from selling birds depends on the price (x), in thousand shillings per bird and is modeled by the polynomial function $R(x) = -2x^2 + 20x$

Where; x –the price per bird in thousand shillings.

$R(x)$ –the total revenue in thousand shillings.

TASK:

As a student of mathematics student of s.5, help the club to;

- Know what type of polynomial function is $R(x) = -2x^2 + 20x$
- Calculate the total revenue when $x = 5$, $x = 10$, $x = 15$
- What price per bird will maximize the total revenue.
- What is the maximum total revenue they can earn?

Item 40

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One Saturday you were having good time driving on a good road with your dad, he drives at a constant speed of **15m/s** for **300** seconds and then he received a call about your uncle waiting at home so he accelerated the car uniformly to a speed of **25m/s** over a period of **20** seconds. This speed is maintained for **300** seconds before the car is brought to rest in **30** seconds at home. He was not supposed to exceed average speed limit of **21m/s** for overall journey otherwise he is to pay a fine of shillings **20,000** to UNRA

Task

As a mathematics student, use graphical analysis to represent the journey described above and use it to find:

- (i) the acceleration
- (ii) the total distance travelled in the time described,
- (iii) whether he is supposed to pay the fine
- (iv) How much was this journey in terms of fuel, if 1 metre = **ugx sh 5**

Item 41

The "Hope for All" NGO is assisting a local health clinic in lira city, Northern Uganda, with their annual covid-19 vaccination drive. They want to understand the reach and effectiveness of their outreach efforts. They collected data from a sample of **40** residents who were present for vaccinations on a particular day and they wanted to do some data analysis and close if the most likely age of people vaccinated is more than **54 years**

Age	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No of residents	4	6	3	5	7	8	5	2

Task

- (a) Assist in-charge in determining the average age and standard deviation of the age of the members who got vaccinated from COVID-19
- (b) Present a graphical analysis to determine whether they closed at the right time

Item 42

Your brother is a director of one nursey school in the city. Eight teachers were short listed for both written and oral interviews. You were invited to be present at the board room, their scores are given below

Written(x)	55	54	35	62	87	53	71	50
Oral (x)	57	60	47	65	83	56	74	43

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Task, As a student of mathematics,

- (a) Represent their scores graphically and comment on the relationship between the two assessments.
- (b) If the successful candidate should score a minimum of 70 in written interview, what should be the minimum score for the oral interview
- (c) Use the data above to test significance at **5%**

Item 43

A city planner is tasked with surveying a triangular park. The three points of the park are given as follows:

The entrance of the park: A(2, 3), fountain at the corner B(8, 11) and a park canteen at C(10, 5). The city planners need to know the area of the whole park for budgetary terms as each 5 units squared Ugx.500,000 has been planned for maintenance.

As well determine the sign post location which should be at the mid-section between the park entrance and canteen.

While at the park students are demonstrating a science project with two robots T-rex and Amd where T-rex moves through points (3,8) and (6,10) and Amd passed through point (4, 4) with its line of movement perpendicular to the direction of T-rex. If they meet at a point K.

Task:

Help the city planner to determine;

- a) Area of the park and how much is to be kept for maintenance
- b) At what coordinates should the sign post be put and how far will the sign post be from the fountain at the corner
- c) Determine the equations of lines describing movement of the robots and hence find the coordinates for point K where they meet

Item 44

Kawalya is designing a triangular garden plot in his backyard. He wants to calculate the area of the garden, analyze how sunlight hits it throughout the day using a model, and simplify expressions related to the light distribution over the garden using trigonometric identities. The triangular garden is formed by three points: A, B, and C. Maya measures the lengths of two sides and the included angle as follows:

- $AB=12$ meters
- $AC=15$ meters
- $\angle BAC=65^\circ$

The sunlight intensity at a certain location in the garden over time is modeled by the function:

$$I(t) = 5 \sin t + 12 \cos t \text{ where } t \text{ is the time in hours after sunrise.}$$

Maya is also working with the expression $\sin A + \sin B$ where $A = 35^\circ, B = 65^\circ$, and she wants to simplify it using trigonometric identities.

Task:

- Using the given dimensions, calculate the area of triangle ABC. Give your answer to 2 decimal places.
- Express the function $I(t)$ in the form $R \sin(t + \alpha)$ and hence interpret what the value of represents in the context of sunlight.
- Use the sum-to-product formula to simplify the expression $\sin 35 + \sin 65$ into a product hence calculate the simplified result and verify that it matches the sum of the original sine values (to 3 decimal places).
- Explain why such identities can be useful in modeling periodic phenomena like sunlight intensity.

Item 45

In certain school, a sick student who is being monitored by nurse on the dosage for malaria

500ml drip entering his body with a modelled equation $X = 500(1 - r)^{\frac{3}{2}t}$ with the rate of 34% and t is the time in hours. She plans to change to an antibiotic when 20ml of the drip is left.

Task:

- If she checks on the student after 50 *minutes*, calculate how much of the drip is left in ml?
- When would she need to change to antibiotic given the drip was initially administered at 08:23 am in the morning?

Item 46

In an ecological study focused on movement of a special new insect species has been identified to be moving following patterns with the equation;

$$24 \operatorname{cosec} \theta = \frac{56}{x^2} \text{ and } \frac{2}{3}y + 42 \sin \theta = 25$$

The model for the whole study program to be a success is based on notation of verifying the following

$$\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 2 \operatorname{cosec} x$$

Task:

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a) Show that the Cartesian equation of motion of the insect is

$$y = \frac{1}{2}(75 - 54x^2)$$

b) Verify that the study program is a success.

Item 47

In certain foreign exchange market model, there two possibilities, either a profit or a loss at any given time, one of the analysts has been tasked to work out the parameter for positive return on investment by solving $\frac{x+3}{x-2} > \frac{x+1}{x-3}$, with x representing risks.

Looking at the future investments in A.I a model for the industry was develop as a whole fraction. As shown $\frac{3x^2+2x+3}{(x+1)(x^2+3)}$. With its possible parts representing market share for USA and China

Task: Help the analyst to:

- (i) Find the regions of pure risk
- (ii) Identify the partials that represent market share.

Item 48

As part of preparation for African Olympiad mathematicians' program one students has come to an S5 principal mathematics student to help solve the following questions for his showing him all relevant steps

Question 1: solve $8^{x-y} = 4^{x+y}$ and $5^{x^2-y^2} = 15625$

Question 2: Show that $\log_5 21 = \frac{1}{2b}(2ab - 2b + a)$ if $a = \log_5 35$ and $b = \log_9 35$

Task: Help the show how this can be solved

Item 49

A boat is sailing directly towards a cliff. The angle of elevation of a point on the top of the cliff and straight a head of the boat increases from 10° to 15° as the ship sails a distance of 50 metres.

A ball is kicked and just lands on a roof 5m high. Using some basic knowledge to find the angle it was kicked at, θ , Ahmed correctly comes up

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with a formula $2 = 4 \tan \theta - \sec^2 \theta$ he is curious about the angle the ball was kicked at. with $0 \leq x \leq 360^\circ$

Task:

- Show that the height of the cliff is approximately is $h \approx 25.783$ metres.
- Help determine the possible angles.

Item 50

You recently got a job as a senior marketing manager at JMT publishers company limited. The induction and public relations team informed you about the major role of boosting sales in the company and increasing on the online visibility of the company. The company executive director gave you data about the number of books sold and corresponding Television and radio adverts made in the year 2024. The data is summarized in the table below

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of adverts(x)	45	56	65	35	72	90	60	79	90	34	56	65
Number of books sold(y)	55	60	75	42	88	98	72	86	88	41	64	78

Note: This data was captured basing on the copies sold to the

bookshops. Tasks;

- As a new marketing manager, advise the top management about the correlation between the adverts and the sales.
- Represent the data on a relevant graph and establish the relationship between the adverts and the sales of the company. If your target is to sale 120 books, how many adverts should the company make? What about if only 20 adverts were made, how books are likely to be sold?
- The quality assurance team would wish to understand the significance of advertising at 10% level, using your expertise, guide the team.

Item 51

Nakato is a mobile money agent in her village, Mukono. When she started in January 2024, she had 50 regular customers. She observed that her customer base seemed to grow exponentially each month. By the end of March 2024 (after 3 months), she had 135 regular customers. She wants to predict her customer growth to plan for liquidity (cash and e-float) and potentially hire an assistant. Assume the growth follows the model $N = N_0 \times K^t$, where N is

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the number of customers after t months, N_0 is the initial number of customers, and K is the monthly growth factor.

Tasks:

- Help Nakato in determining her monthly growth factor, K . Express your answer to 3 significant figures.
- Determine the number of customers Nakato can expect by the end of December 2024 (after 12 months) if this growth rate continues.
- Nakato estimates she needs UGX 10,000 in float per regular customer per month. Using the predicted number of customers for December 2024, calculate the total float she would need. Express this amount using index notation in terms of powers of 10.

Item 52

Mr. Okello is weaving a traditional Ugandan mat (ekikeeka) with intricate geometric patterns. One key element involves fitting square tiles made of dyed reeds into a rectangular border. The side length of each square tile needs to be exactly $(\sqrt{5} - \sqrt{2})m$ for the pattern to align perfectly. The rectangular border has a length of $(10\sqrt{5} + 5\sqrt{2})m$ and a width of

$(8\sqrt{5} - 4\sqrt{2})cm$.

Tasks:

- Calculate the exact area of one square tile. Express your answer in the simplest surd form
 $a + b\sqrt{c}$.
- Determine the exact area of the rectangular border. Express your answer in the simplest surd form.
- If Mr. Okello wants to fit as many *whole* square tiles as possible within the border without overlapping, estimate the maximum number of tiles he can fit. Justify your answer.

Item 53

A local environmental group in Jinja is studying the population growth of a specific fish species in a protected section of the Nile River. Their initial estimate in 2020 was 1,200 fish. They believe the population P after t years can be modelled by $(t) = 1200 \times (1.15)^t$. However, another model proposed is based on logarithms: $\log_{10} P = \log_{10} 1200 + t \log_{10} 1.15$.

Tasks:

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- a) Using the index model $(t) = 1200 \times (1.15)^t$, calculate the predicted fish population in the year 2025.
- b) Using the logarithmic model, show that it is equivalent to the index model. c)
- d) The group wants to know when the fish population is predicted to reach 5,000. Using logarithms and the model $(t) = 1200 \times (1.15)^t$, determine the approximate number of years (t) it will take.

Item 54

Mrs. Nabukalu, a farmer in Masaka, finds that the yield of her maize crop (in bags per acre), Y , depends on the amount of a specific fertilizer x used (in kg per acre). The relationship is modelled by the quadratic equation: $(x) = -0.5x^2 + 20x + 50$. She wants to maximize her yield but also knows the fertilizer costs money.

Tasks:

- a) Help Mrs. Nabukalu to determine the amount of fertilizer (x) that will give her the maximum maize yield.
- b) Calculate the maximum possible yield in bags per acre.
- c) If the cost of the fertilizer is UGX 1,500 per kg. Mrs. Nabukalu wants the yield to be at least 200 bags per acre. Formulate a quadratic inequality to represent this situation.
- d) By solving the inequality in part (c) above determine the range of fertilizer amounts (in kg per acre) she can use to achieve a yield of at least 200 bags per acre.

Item 55

A community group in Gulu is managing the costs for drilling three boreholes (A, B, and C). The total cost was UGX 25,000,000. The cost of Borehole B was UGX 1,000,000 less than Borehole A. The combined cost of Boreholes A and C was three times the cost of Borehole B. Let the costs of drilling boreholes A, B, and C be a , b , and c (in UGX) respectively.

Tasks:

- a) Formulate a system of three linear simultaneous equations representing the information given.

- b) By Using Row reduction, calculate the individual cost of drilling each borehole (a, b, and c).
- c) If the cost per meter drilled was UGX 250,000 for all boreholes, determine the depth of Borehole A.

Item 56

A school in Mbarara wants to create a rectangular vegetable garden. They have 80 meters of fencing available. They want the area of the garden to be greater than 300 square meters to grow enough vegetables for the school lunch program. Let the length of the garden be L meters and the width be W meters.

Tasks:

- a) Express the perimeter of the garden in terms of L and W and form an equation using the available fencing.
- b) Using your equation in a) above formulate the area A of the garden purely in terms of L .
- c) Formulate a quadratic inequality representing the condition that the area must be greater than 300 square meters and solve it to determine the possible range of values for the length (L) of the garden that satisfies both the fencing constraint and the area requirement.

Item 57

Mr. Kato owns a rectangular farm near Fort Portal. On a map grid, the corners of his main plot are at $(1, 2)$, $B(9, 2)$, $C(9, 8)$, and $D(1, 8)$. He plans to install two straight irrigation pipes. *Pipe 1* will run from corner A to corner C. *Pipe 2* will run from the midpoint of side AB to the midpoint of side CD. A water source is located at point W $(5, 5)$.

Tasks:

- a) Determine the coordinates of the midpoints of sides AB and CD.
- b) Formulate the equation of the line representing Pipe 1 (line AC) and the equation of the line representing Pipe 2.
- c) Determine the shortest distance from the water source W $(5, 5)$ to Pipe 1 (line AC). Will Pipe 1 pass directly through the water source? Justify your answer.

Item 58

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A new road (Road 1) is being constructed in Kampala, represented by the equation $y = 2x + 3$. It will intersect an existing road (Road 2), represented by the equation $3x + 2y = 12$. A traffic light needs to be installed at the intersection point. Another planned road (Road 3) needs to be parallel to Road 1 and pass through the point $P(4, 1)$. A fourth road (Road 4) must be perpendicular to Road 2 and pass through the same point $(4, 1)$.

Tasks:

- Calculate the coordinates of the intersection point of Road 1 and Road 2 where the traffic light will be placed.
- Determine the equation of the line representing the planned Road 3 and Road 4.
- Calculate the acute angle between Road 1 and Road 2 at their intersection point. Give your answer in degrees.

Item 59

Three villages, A, B, and C, are located on a map grid at coordinates $A(2, 1)$, $B(8, 3)$, and $C(4, 7)$. A new health centre needs to be built such that it is equidistant from villages A and B. It must also lie on the line that passes directly between village C and the midpoint of the line segment connecting A and B.

Tasks:

- Determine the coordinates of the midpoint M of the line segment connecting villages A and B.
- Formulate the equation of the perpendicular bisector of the line segment AB. (This line represents all points equidistant from A and B).
- Find the equation of the line passing through village C $(4, 7)$ and the midpoint M calculated in Task 1.
- Calculate the coordinates where the two lines found in b) and c) intersect. This point represents the ideal location for the health centre. Justify why this location satisfies both conditions.

Item 60

An Engineer in a chemical engineering plant in Namanve, want to use a chemical with concentration (t) of a product over time t , which is modelled by complex rational functions. Suppose the rate of change of concentration involves the expression: $f(t) = \frac{5t + 3}{(t+1)(t+2)}$. To

analyse the long-term behaviour but doesn't know the appropriate techniques to use. **Tasks:**

- Help the engineer to identify the type of factors in the denominator of $f(t)$.
- Express $f(t)$ as the sum of its partial fractions.

Item 61

An electrical engineering student at Makerere University is analysing a signal whose behaviour

over time x is related to the function $g(x) = \frac{2x^2 + x - 1}{x(x-1)^2}$. This expression needs to be broken down

for further analysis.

Tasks:

- Set up the appropriate form for the partial fraction decomposition of $g(x)$.
- Determine the values of the unknown constants in the partial fraction decomposition.
- Write the final partial fraction decomposition of $g(x)$.

Item 62

An economist is studying the relationship between investment I and national income Y . The relationship involves a complex function where a particular term is given by $h(Y) = \frac{Y^3 + 2Y^2 - Y + 5}{Y^2 + Y - 2}$. Before proceeding with the economic analysis, the economist needs to simplify this expression.

Tasks:

Help the economist to;

- Identify $h(Y)$ as a proper or improper rational function. Justify your answer.
- express $h(Y)$ as the sum of a polynomial and a proper rational fraction.
- Take the proper rational fraction part obtained in b) and decompose it into its partial fractions.
- Combine the results from b) and c) to write the complete simplified expression for $h(Y)$.

Item 63

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A surveyor is mapping a triangular piece of land in the hilly region of Kabale. The vertices are marked as P, Q, and R. The distance PQ is measured as 120 meters, and the distance PR is 150 meters. The angle at P, $\angle QPR$, is measured as 75° . The surveyor needs to find the length of the third side QR and the area of the land.

Tasks:

- Determine the length of the side QR to the nearest meter and angle $\angle PQR$ to 1 decimal place.
- Calculate the area of the triangular piece of land PQR.
- If $\angle QPR$ was actually measured as $(45^\circ + 30^\circ)$, apply an appropriate formula find the exact value of $\cos 75^\circ$.

Item 64

An architect is designing a symmetrical roof truss for a community hall in Lira. The truss is shaped like an isosceles triangle ABC, with $AB = AC$. The base BC has a length of 16 meters. The angle at the apex A, $\angle BAC$, needs to be determined such that the height (altitude from A to BC) is exactly 6 meters. Let M be the midpoint of BC.

Tasks:

- Consider the right-angled triangle AMB. Calculate the length of the side AB
- In triangle AMB, determine the value of $\tan(\angle ABM)$ and hence find $\angle ABM$ in degrees.
- determine the measure of $\angle BAM$ hence calculate the angle at the apex, $\angle BAC$.

Item 65

A fishing boat leaves Kasenyi landing site (Point K) and travels 15 km on a bearing of 060° to reach Point A. From Point A, it then travels 20 km on a bearing of 135° to reach Point B. The boat captain now wants to know the direct distance and bearing from Kasenyi (K) back to Point B.

Tasks:

- Help the captain to map the journey on a diagram, showing the points K, A, B, and their bearings.
- By applying cosine rule calculate the direct distance KB, correct to one decimal place.
- By applying sine rule calculate the angle $\angle AKB$. Hence, determine the bearing of Kasenyi (K) from Point B.

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Item 66

A cooperative society of farmers in Luwero recorded the cassava yield (in tonnes per hectare) from 50 small plots. The data is grouped as follows:

Yield (Tonnes/Hectare)	Number of Plots (Frequency)
5 - < 10	6
10 - < 15	10
15 - < 20	15
20 - < 25	11
25 - < 35	8

Tasks:

- Construct a histogram to represent this data.
- Using the histogram, estimate the modal yield of cassava per hectare.
- Calculate an estimate of the mean yield and the standard deviation of the yield for these plots.

Item 67

The scores of 80 Senior Five students in a Mathematics mock exam at a school in Arua are summarised in the following cumulative frequency table:

Score (x)	Cumulative Frequency
$x \leq 20$	5
$x \leq 30$	15
$x \leq 40$	35
$x \leq 50$	55
$x \leq 60$	70
$x \leq 70$	77
$x \leq 80$	80

Tasks:

- Represent the data on a cumulative frequency curve (ogive) to represent this data and use it to estimate:
 - The median score.

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- ii) The interquartile range of the scores.
 - iii) The 80th percentile score.
- b) If the pass mark was set at 45 marks, estimate from your ogive the number of students who passed the exam.
- c) Explain what the interquartile range tells you about the spread of the students' scores.

Item 68

Two market vendors, Aisha and Ben, operating in Owino Market, Kampala, recorded their daily sales (in thousands of UGX) over a period of 30 days. The data is summarized below:

Aisha: Mean Sale = 150, Standard Deviation = 25

Ben: Mean Sale = 160, Standard Deviation = 40 **Tasks:**

- a) Determine which vendor has higher average daily sales.
- b) Calculate the coefficient of variation for both Aisha and Ben.
- c) Using the coefficient of variation, determine whose sales are relatively more consistent.

Justify your answer.

Item 69

An agricultural officer in the Bugisu region collects data on the annual rainfall (in mm) and the coffee yield (in kg per tree) for 8 different farms over the past year.

Rainfall (mm), x	Yield (kg/tree), y
1200	2.5
1400	3.0
1000	2.0
1600	3.2
1800	3.5
1100	2.2
1500	3.1
1300	2.8

Tasks:

- Construct a scatter diagram to visually represent the relationship between rainfall and coffee yield.
- Based on the scatter diagram, describe the type of correlation you observe between rainfall and yield.
- By ranking the data for both rainfall (x) and yield (y), calculate Spearman's rank correlation coefficient.
- Interpret the value of Spearman's rank correlation coefficient you calculated in the context of rainfall and coffee yield in this region. Does it support your observation from the scatter diagram?

Item 70

A teacher at Exodus College School wants to investigate if there's a relationship between the average number of hours students spend studying per week and their score on a recent Physics test. Data for 7 students is collected:

Study Hours/Week (x)	Test Score (y)
5	65
8	75
2	50
10	85
4	60
12	90
6	72

Tasks:

- Help the teacher to represent this data on a scatter diagram.
- Visually, draw a line of best fit through the points on your scatter diagram.
- Comment on the apparent relationship between study hours and test scores based on your diagram and line of best fit.
- Would it be reasonable to use this relationship to predict the score of a student who studies for 20 hours a week? Explain your reasoning, considering the limitations of extrapolation.

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Item 71

An economics student is researching the relationship between the average weekly price of a bunch of Matooke (in UGX) in Nakasero market and the estimated quantity demanded (in hundreds of bunches). Data over 6 weeks is collected:

Price (UGX), P	Quantity (hundreds), Q
5000	80
6000	70
4500	90
7000	60
5500	75
Price (UGX), P	Quantity (hundreds), Q
6500	68

Tasks:

- Calculate the Spearman's rank correlation coefficient between the price and quantity demanded.
- Interpret the calculated correlation coefficient. Does it align with typical economic principles of demand?
- Plot a scatter diagram for the Price (P) vs Quantity (Q). Does the visual pattern support the calculated correlation?

Item 72

Two farm workers, Okello and Lanyero, are pulling a heavy sack of maize (mass 80 kg) across level ground in a Kireka warehouse. Okello pulls with a force of 300 N at an angle of 20° above the horizontal. Lanyero pulls with a force of 250 N at an angle of 15° above the horizontal, in the same direction as Okello. The coefficient of kinetic friction between the sack and the ground is

0.3. (Assume $g = 9.8 \text{ m/s}^2$).

Tasks:

- Represent all the forces acting on the sack on a diagram.
- Resolve the forces applied by Okello and Lanyero into horizontal and vertical components.

- c) Calculate the total upward vertical component from the workers' pulls and hence determine the Normal Reaction force exerted by the ground on the sack.
- d) Calculate the maximum possible frictional force and the total horizontal component of the pulling forces hence determine the net horizontal force acting on the sack.

Item 73

In a mechanics lab, a block A of mass 5 kg rests on a rough inclined plane angled at 30° to the horizontal. The coefficient of kinetic friction between block A and the plane is 0.2. Block A is connected by a light inextensible string passing over a smooth pulley at the top of the incline to a block B of mass 3 kg, which hangs freely. The system is released from rest. (Assume $g = 9.8 \text{ m/s}^2$).

Tasks:

- a) Illustrate the forces acting on block A and block B on separate diagrams.
- b) For block A, resolve its weight into components parallel and perpendicular to the inclined plane. Calculate the normal reaction force on block A.
- c) Determine the frictional force acting on block A as it slides (assume it slides up the plane initially, if unsure, calculate net force without friction first to determine direction).
- d) Apply Newton's Second Law to both block A and block B to formulate two simultaneous equations involving the acceleration (a) of the system and the tension (T) in the string.

Solve these equations to find the values of a and T .

Item 74

A lorry of mass 5000 kg is parked on a road in Kisoro inclined at an angle θ to the horizontal, where $\sin \theta = 0.1$. The coefficient of static friction between the lorry's tyres and the road is 0.4.

The driver has applied the handbrake. We want to determine if the lorry will remain stationary.

(Assume $g = 9.8 \text{ m/s}^2$).

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Tasks:

- a) Come up with a diagram showing the forces acting on the lorry assuming it is about to slide down the slope.
- b) Resolve the weight of the lorry into components parallel and perpendicular to the road surface, hence find the Normal Reaction force acting on it.
- c) Determine the maximum possible static frictional force that can be exerted by the road on the tyres ($F_{\max} = \mu_s N$). Compare this maximum friction with the component of the lorry's weight acting down the slope. Establish whether lorry will remain stationary or slide down. Justify your conclusion.

Item 75

In a certain region of Uganda, it is estimated that 2% of the population has a particular disease. A medical test is developed to detect the disease. The test is not perfect:

If a person has the disease, the test correctly gives a positive result 95% of the time (Sensitivity).

If a person does not have the disease, the test correctly gives a negative result 90% of the time (Specificity). A person from the region is selected at random and tested.

Tasks:

- a) Construct a tree diagram and use it to calculate the overall probability that a randomly selected person tests positive.
- b) Using Bayes' Theorem, determine the probability that a person actually has the disease given that they tested positive.
- c) Interpret your result from b) above. What does this tell you about the reliability of a positive test result in this scenario?

Item 76

A factory in Jinja produces light bulbs using three machines: Machine A, Machine B, and Machine C.

Machine A produces 40% of the total output, and 5% of its bulbs are defective.

Machine B produces 35% of the total output, and 3% of its bulbs are defective.

Machine C produces 25% of the total output, and 2% of its bulbs are defective.

A bulb is selected at random from the factory's output.

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Tasks:

- a) Determine the probability that the selected bulb was produced by Machine A AND is defective. Similarly, calculate the probabilities for Machine B being defective and Machine C being defective.
- b) Using the results from a), determine the overall probability that a randomly selected bulb from the factory's output is defective.
- c) Given that the selected bulb is found to be defective, calculate the probability that it was produced by Machine B.

Item 77

In a class of 60 students at a Kampala school, 40 own an Android phone (A), 25 own an iPhone (I), and 15 own both types.

Tasks:

- a) Represent this information on a Venn diagram.
- b) Determine the number of students who own:
 - i) Only an Android phone.
 - ii) Only an iPhone.
 - iii) Neither type of phone.
- c) A student is selected at random from the class. Calculate the probability that the student owns:
 - i) An Android phone or an iPhone.
 - ii) Exactly one type of phone.
- d) Given that a selected student owns an Android phone, calculate the probability that they also own an iPhone.

Item 78

A farmer in Mukono wants to create a rectangular enclosure for chickens next to a long, straight existing wall. He has 100 meters of fencing wire available for the other three sides of the rectangle. He wants to maximize the area enclosed for his chickens. Let the side parallel to the wall have length x meters, and the other two sides perpendicular to the wall have length y meters each.

Tasks:

- Help the farmer to express the total length of the fencing used in terms of x and y , and formulate an equation based on the available wire.
- Express the area A of the enclosure ($A = xy$) as a function of only one variable x .
Hence, find the value of x that maximizes the area.
- Determine the maximum possible area of the enclosure and confirm it is a maximum.

Item 79

The displacement s (in meters) of a particle moving along a straight line from a fixed point O , at time t (in seconds), is given by the equation $s(t) = t^3 - 6t^2 + 9t + 5$, for $t \geq 0$.

Tasks:

- Determine the expressions for the velocity $v(t)$ and acceleration $a(t)$ of the particle at time t by differentiating the displacement function.
- Calculate the initial velocity and initial acceleration of the particle at $t = 0$.
- Find the time(s) when the particle is momentarily at rest ($v(t) = 0$).
- Determine the acceleration of the particle at the time(s) when it is at rest. Describe the motion of the particle during the first 4 seconds.

Item 80

A scientist has a spherical balloon which is being inflated. Its radius r is increasing at a constant rate of 0.1 cm per second. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. He wants to find the rate at which the volume is increasing when the radius is 5 cm. He also wants to estimate the approximate increase in volume as the radius increases from 5 cm to 5.1 cm.

Tasks:

- Help the scientist to determine the rate at which the volume of the bowl is changing with respect to the radius.
- Determine the rate at which the volume is increasing when the radius $r = 5$ cm.
- Estimate the approximate increase in volume (δV) as the radius increases from $r = 5$ cm to $r = 5.1$ cm.

Item 81

Water flows into a storage tank in Mbale at a rate given by $(t) = 10 + 0.5t$ liters per minute, where t is the time in minutes from the start ($t = 0$). The tank was initially empty.

Tasks:

- Obtain expression for the volume $V(t)$ of water in the tank as an indefinite integral.
- Using the initial condition, determine the value of the constant of integration C .
- Calculate the volume of water in the tank after 60 minutes
- Determine the average rate of flow into the tank during the first 60 minutes using the mean value function.

Item 82

A piece of land is bounded by a river whose shape can be modelled by the curve $y = \sqrt{x}$, the straight line $x = 9$, and the x-axis (representing a straight fence). The coordinates are measured in meters. The owner wants to calculate the area of this piece of land.

Tasks:

- Help the owner to sketch the region bounded by the land.
- Set up the definite integral that represents the area of this region hence use it to calculate the exact area of the piece of land.
- If this area were revolved around the x-axis, it would form a solid shape. Set up the definite integral representing the volume of this solid of revolution hence Calculate this volume.

Item 83

A boda-boda rider accelerates away from a traffic light in Fort Portal. His velocity v (in m/s) after time t (in seconds) is given by $(t) = 6t - t^2$ for $0 \leq t \leq 6$.

Tasks:

- Obtain an expression for his displacement (t) (assume $s(t = 0) = 0$).
- Determine the displacement of the boda-boda from the traffic light after 3 seconds.
- Calculate the total distance travelled by the boda-boda in the first 6 seconds.

- d) Determine the time t at which the boda-boda reaches its maximum velocity within the interval $0 \leq t \leq 6$. Hence Calculate its maximum velocity.

Item 84

A student at Ntare School has 4 distinct Mathematics books, 3 distinct Physics books, and 2 distinct Chemistry books. He wants to arrange them on a single shelf.

Tasks:

Help the student to know

- how many different ways he can arrange 9 books be on the shelf if there are no restrictions?
- how many ways he can arrange the books if all the Mathematics books must be kept together, all the Physics books must be kept together, and all the Chemistry books must be kept together?
- In how many ways he can arrange the books if only the Mathematics books must be kept together?

Item 85

Exodus College School needs to form a student committee of 5 members. There are 8 eligible students from Senior Five and 6 eligible students from Senior Six.

Tasks:

- In how many ways can the committee of 5 be formed if there are no restrictions on the class level?
- In how many ways can the committee be formed if it must consist of exactly 3 students from Senior Five and 2 students from Senior Six?
- In how many ways can the committee be formed if it must include at least 4 students from Senior Five?
- Suppose two specific Senior Six students, Mary and Jane, refuse to be on the committee together. In how many ways can the committee be formed if it must have exactly 3 Senior Five students and 2 Senior Six students, considering this restriction?

Item 86

A mobile banking App requires users to create a 4-digit PIN using the digits 0 to 9.

Tasks:

- a) How many different 4-digit PINs can be created if digits can be repeated?
- b) How many different 4-digit PINs can be created if digits cannot be repeated?
- c) How many different 4-digit PINs can be created if digits cannot be repeated and the PIN must be an even number?
- d) How many different 4-digit PINs can be created if digits can be repeated, but the PIN cannot start with 0?

Item 87

Your teacher presents a challenging problem of determining the vertical height of the flagpole and the horizontal distance between two students, Mia and Sam, who are looking up at the top of the school flagpole. Mia is 1.5 meters tall and stands 10 meters from the base of the flagpole, measuring an angle of elevation of 35° to the top with a clinometer. Sam stands 15 meters away, measuring an angle of elevation of 25° ; however, his height is unknown, although he is taller than Mia. They are on the same side of the flagpole, in a straight line from its base, indicating that Sam is 5 meters farther from the pole than Mia. For safety reasons, the school plans to install a safety net under the flagpole to catch the flag if it falls during the daily flag-raising ceremony, ensuring student safety while maintaining a clean yard. The cost of the net is UGX 50,000 for each meter of the flagpole's height. Both students are on flat ground.

Task: Help the teacher calculate the cost of the safety net and determine Sam's height.

Item 88

It was Sports Day at your school, where students cheered from the sidelines as athletes ran on the field. Three students, Amara, Ben, and Chloe, members of the entrepreneurial club, set up a snack stall near the sidelines to sell to the cheering crowd. Amara made chapatis, Ben made samosas, and Chloe made mandazis. They competed to see who could sell the most and earn the highest sales. However, the busy day created a challenge when they attempted to record only the total sales in their notebooks, instead of the price of each item. Their first sale was to the cheering squad: six chapatis, ten samosas, and three mandazis for UGX 10,500. At midday, the second sale was to the teachers: two chapatis, eight samosas, and seven mandazis for UGX 7,900. Later, the third sale of the day was to the football team: five chapatis, four samosas, and four mandazis for UGX 8,200. When Sports Day ended, Amara,

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Ben, and Chloe sat with their notebooks. They wanted to find out the prices of one chapati, one samosa, and one mandazi to see who won the competition and plan for the stock for the next day.

Task: Who won the competition based on sales amount?

Item 89

Onyumel runs a maize flour business near Jinja Market. To minimize stock costs, he cultivates maize on a farm close to the Source of the Nile. His initial rectangular plot measures 120 square meters, which is sufficient for his needs, featuring a length that is 5 meters longer than the width, allowing for orderly rows. Each square meter produces 2 kg of maize per season. Seeds are priced at UGX 5,000 per kilogram, with 1 kilogram covering an area of 10 square meters. Fencing costs UGX 1,500 per meter, and he has allocated a budget of UGX 90,000 to keep animals at bay. As demand increases, he plans to expand the land area to 150 m², maintaining a length of 5 meters and a width greater than 5 meters. He considers the implications if fencing prices rise to UGX 2,000 per meter.

Task:

- (a) How many kilograms of maize are planted in the expanded plot?
- (b) What is the difference in the cost of fencing the expanded plot and the first plot?

END
Good luck

**SUCCESS IN YOUR ACADEMIC
JOURNEY**

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